# WAITING FOR DROUGHT

## A Renewal Process Approach to Modeling Inter-arrival Times between Drought Onsets

Chao Han (with Dong-yun Kim)
Department of Statistics, Virginia Tech

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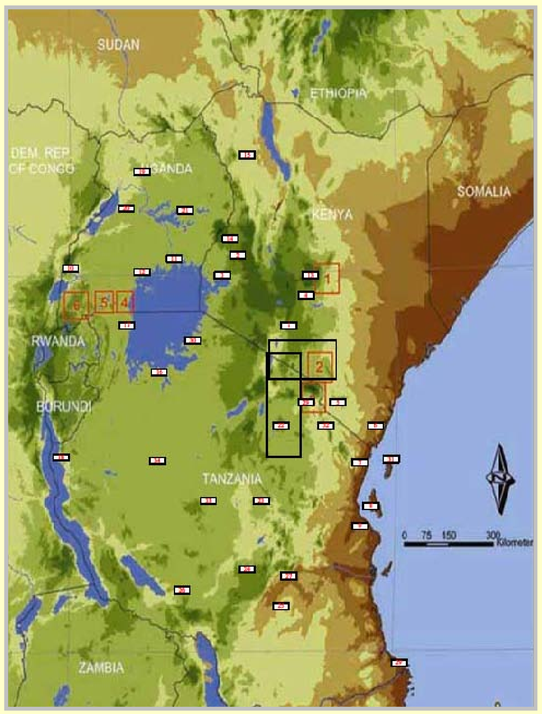
## INTRODUCTION

Drought is a regular and recurrent event in climate of East Africa which affects livestock, crops and human lives in the region. Accurate description of drought onset pattern is important for farmers and planners. The research efforts on the stochastic approach have not been very extensive and mainly concerned with drought duration. Not much work has been done focus on drought frequency.

We suggest a Poisson process approach for building a probabilistic model of inter-arrival times between successive onsets of drought. The method is illustrated using the Standardized Precipitation Index (SPI) based on monthly precipitation data from 35 stations across Kenya, Tanzania, and Uganda (Figure 1). It is noted that the homogeneous Poisson process explains the data quite well, and our approach can be applied to study regional characteristics regarding drought onset frequency.

**Figure 1** *Locations of the 35 regions in East Arica where monthly precipitation data during 1900~2000 is collected*




## METHODS

### Standardized Precipitation Index (SPI)

➢Can be calculated for different time scales solely based on precipitation data
➢SPI value indicates deviation from precipitation mean; negative SPI for drought and positive for wet conditions

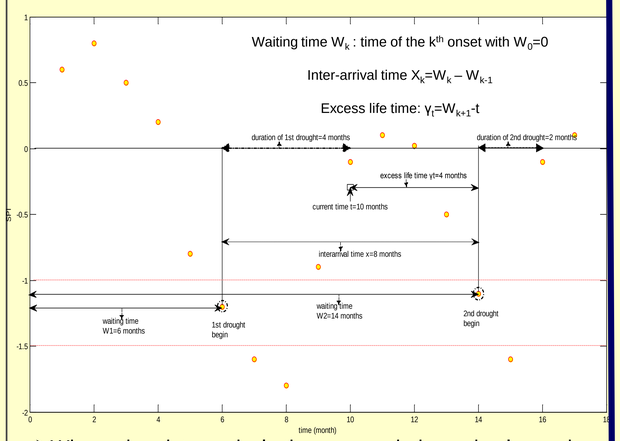
**Table 1** *drought classification according to SPI values and time scales*

| SPI value | Drought Classes |
|---|---|
| 0 to -0.99 | Near normal |
| -1.0 to -1.49 | Moderate drought |
| -1.5 to -1.99 | Severe drought |
| -2.0 or less | Extreme drought |
| **SPI time scale** | **Associated Drought Type** |
| 6-month | Agricultural drought |
| 12-month | Hydrological drought |

### Stochastic model for drought onset

➢Onset of drought as defined by SPI; drought starts when SPI dips below –1 and ends when SPI becomes positive
➢Counting Process N(t): number of onsets up to current time t>0 and N(0)=0

**Figure 2** *Waiting Time, Inter-arrival Time and Excess Life Time*




➢When the inter-arrival times are independently and identically distributed with an exponential distribution with parameter λ, N(t) has a homogeneous Poisson process
➢For the Poisson process, the excess life time $\gamma_t$ possesses the same exponential distribution with the same λ value as the inter-arrival time
➢Model can be used to predict the probability that the next drought will start after x months

$$\Pr(\gamma_t > x) = e^{-\lambda x} \qquad (1)$$

➢λ equals to 1/(mean of the inter-arrival time)

## RESULTS

### Summary of data analysis

➢Transform the historical precipitation data of 35 regions in East Africa to the SPI values at different time scales
➢Calculate all the inter-arrival times between any two successive moderate or severe droughts in both 6-month and 12-month time scale

**Table 2** *Estimated inter-arrival time means (sample average from observed data) for some of the 35 regions*

| Site # | Region Name | Moderate 6 month | Moderate 12 month | Severe 6 month | Severe 12 month |
|---|---|---|---|---|---|
| 1 | Nairobi | 35.59 | 49.88 | 51.13 | 83.5 |
| 2 | Eldoret | 31.5 | 55.5 | 72.6 | 89.2 |
| 8 | Zanzibar | 34.09 | 66.17 | 51.29 | 90 |
| 9 | Dar es Salaam | 28.19 | 46.54 | 52.36 | 72 |
| 12 | Entebbe Airport | 27.16 | 57.32 | 61.58 | 95.08 |
| 17 | Bukoba | 48.22 | 76.64 | 59.28 | 151.43 |

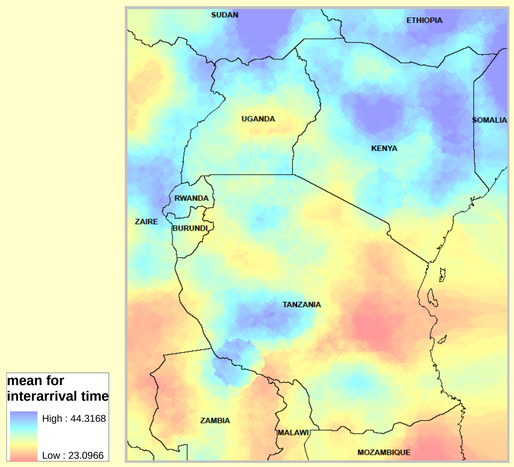
### Model Adequacy and Inference

➢Kolmogorov-Smirnov distribution test and Box-Pierce independent test show Poisson process model is suitable for the drought onset times
➢Predict the probability that the next moderate drought will start after 8 months (with 6-month time scale) for region Nairobi, use formula (1) taking λ=1/mean=1/35.59=0.028

$$\Pr(\gamma_t > 8) = e^{-0.028*8} = 0.799$$

### Spatial drought onset pattern based on inter-arrival average

**Figure 3** *Spatial drought pattern map based on inter-arrival time average of moderate drought (6-month time scale) for Climate Research Unit (CRU) data consisting of 1163 sites in the East Africa region. (***Short inter-arrival time average between two successive drought onsets indicates drought occurs very frequently at this region**)




## CONCLUSIONS

➢The inter-arrival time between droughts in 35 regions can be modeled by a homogeneous Poisson process
➢Our approach can answer questions about the chance that the next drought occurs after a specified time
➢Site-specific data can be used to classify regional drought characteristics
➢In the future, may consider alternating renewal model, compound Poisson process to include drought duration and magnitude together with inter-arrival time.

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## For further information

Please contact dongyunkim@vt.edu
or chaohan@vt.edu

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